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Mathematics: analysis and approaches

Standard level

Paper 1

30 October 2023

Zone A afternoon | Zone B afternoon | Zone C afternoon

Candidate session number

1 hour 30 minutes

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Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



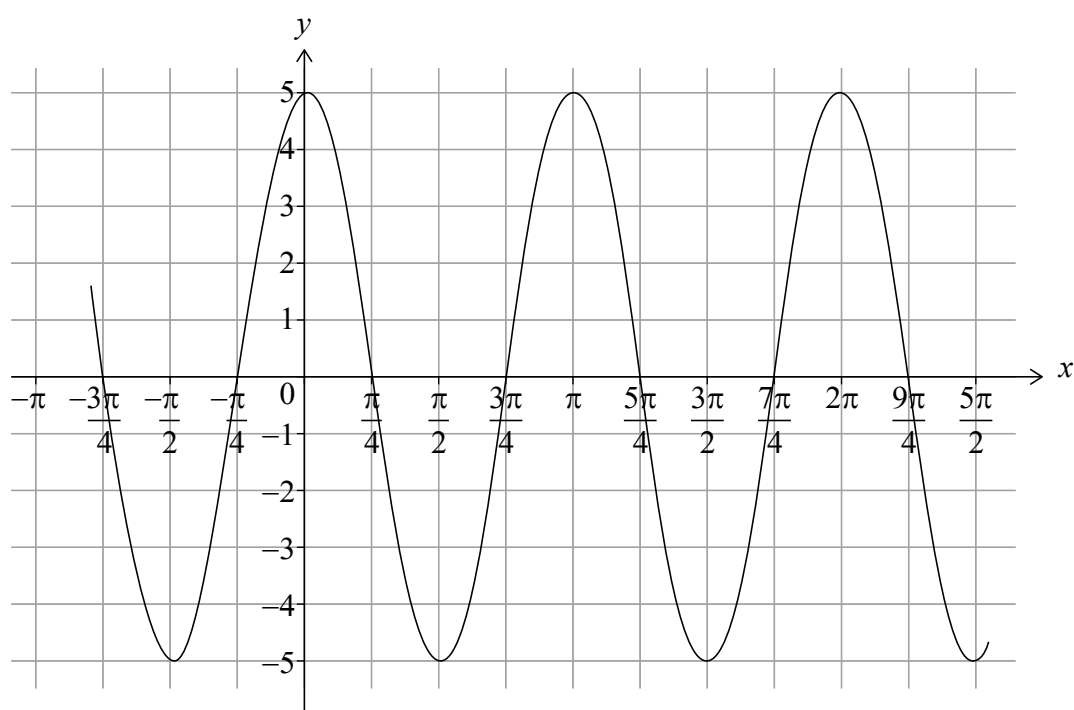
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function $f(x) = a \cos(bx)$, with $a, b \in \mathbb{Z}^+$. The following diagram shows part of the graph of f .



- (a) Write down the value of a . [1]
- (b) (i) Write down the period of f . [3]
(ii) Hence, find the value of b .
- (c) Find the value of $f\left(\frac{\pi}{6}\right)$. [3]

(This question continues on the following page)



(Question 1 continued)

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16EP03

Turn over

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will not be marked.



2. [Maximum mark: 5]

Consider the functions $f(x) = x - 3$ and $g(x) = x^2 + k^2$, where k is a real constant.

(a) Write down an expression for $(g \circ f)(x)$. [2]

(b) Given that $(g \circ f)(2) = 10$, find the possible values of k . [3]

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3. [Maximum mark: 4]

Events A and B are such that $P(A) = 0.65$, $P(B) = 0.75$ and $P(A \cap B) = 0.6$.

(a) Find $P(A \cup B)$. [2]

(b) Hence, or otherwise, find $P(A' \cap B')$. [2]

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4. [Maximum mark: 7]

The sum of the first n terms of an arithmetic sequence is given by $S_n = pn^2 - qn$, where p and q are positive constants.

It is given that $S_4 = 40$ and $S_5 = 65$.

(a) Find the value of p and the value of q . [5]

(b) Find the value of u_5 . [2]

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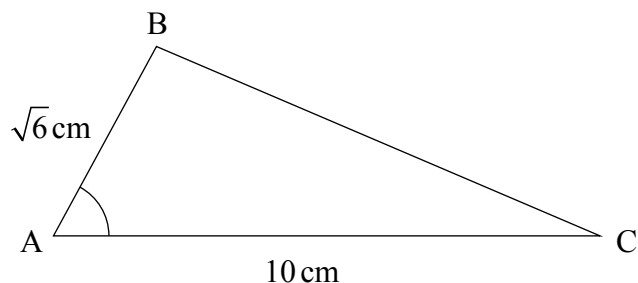
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5. [Maximum mark: 6]

In the following triangle ABC , $AB = \sqrt{6}$ cm, $AC = 10$ cm and $\cos \hat{BAC} = \frac{1}{5}$.

diagram not to scale



Find the area of triangle ABC .

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6. [Maximum mark: 6]

The binomial expansion of $(1 + kx)^n$ is given by $1 + 12x + 28k^2x^2 + \dots + k^nx^n$ where $n \in \mathbb{Z}^+$ and $k \in \mathbb{Q}$.

Find the value of n and the value of k .

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 17]

A ballet company performs *The Nutcracker* every year. Last year they gave a total of 60 performances at their theatre which has a maximum capacity of 800. The number of tickets sold, n , at each performance is shown in the following frequency table.

Number of tickets sold, n	Number of performances
$0 < n \leq 200$	3
$200 < n \leq 400$	p
$400 < n \leq 600$	18
$600 < n \leq 800$	30

(a) (i) Find the value of p .

(ii) Write down the modal class.

[2]

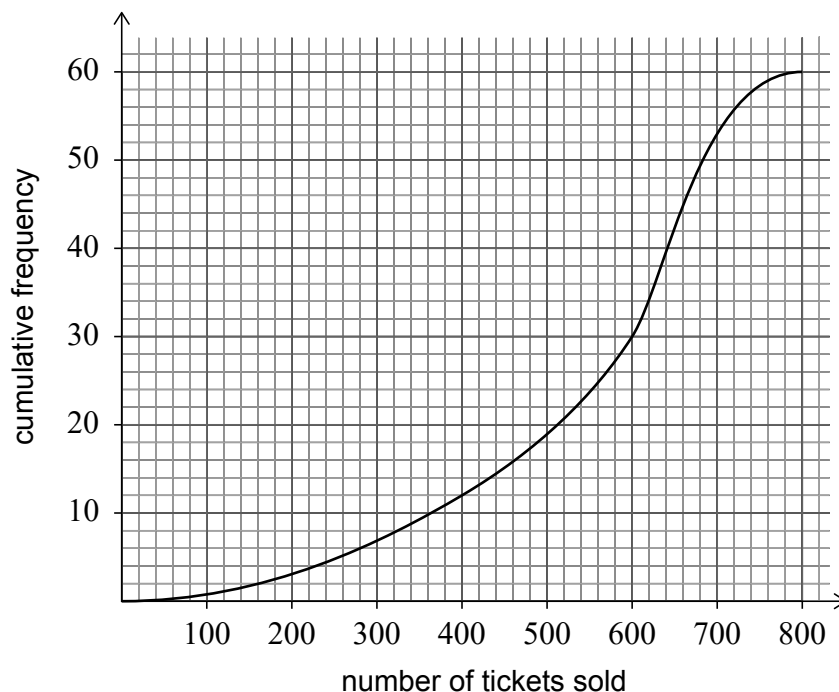
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(Question 7 continued)

The following cumulative frequency diagram also displays these data.



(b) Use the cumulative frequency curve to estimate

- (i) the median number of tickets sold;
- (ii) the number of performances where at least 80% of the tickets were sold.

[4]

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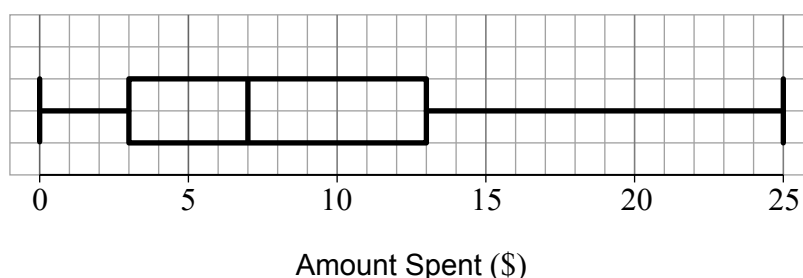
(Question 7 continued)

After a performance, the company decides to conduct a survey to obtain feedback from the audience.

- (c) (i) State one disadvantage of the company surveying only the first 5 % of the audience as they leave the theatre.
- (ii) Describe briefly how the company could collect feedback from 5 % of the audience using the systematic sampling method.
- (iii) State the sampling method which should be used if the survey is to be representative of the number of children and the number of adults in the audience. [4]

Last year 36 000 tickets were sold to *The Nutcracker*.

- (d) The following box and whisker diagram displays the amount spent by the audience at the souvenir shop when they attended the performance.



- (i) Estimate the number of people who spent between \$3 and \$25.
- (ii) Half the audience spent less than \$ a . Estimate the value of a . [3]
- (e) This year the company will again give 60 performances and expects to sell 17 additional tickets for each performance.
- (i) Calculate the mean number of tickets the company expects to sell this year for each performance.
- (ii) State what effect, if any, this increase in ticket sales would have on the variance of the number of tickets sold for each performance. [4]



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8. [Maximum mark: 15]

The functions f and g are defined by

$$f(x) = \ln(2x - 9), \text{ where } x > \frac{9}{2}$$

$$g(x) = 2 \ln x - \ln d, \text{ where } x > 0, d \in \mathbb{R}^+.$$

(a) State the equation of the vertical asymptote to the graph of $y = g(x)$. [1]

The graphs of $y = f(x)$ and $y = g(x)$ intersect at two distinct points.

(b) (i) Show that, at the points of intersection, $x^2 - 2dx + 9d = 0$.

(ii) Hence show that $d^2 - 9d > 0$.

(iii) Find the range of possible values of d . [9]

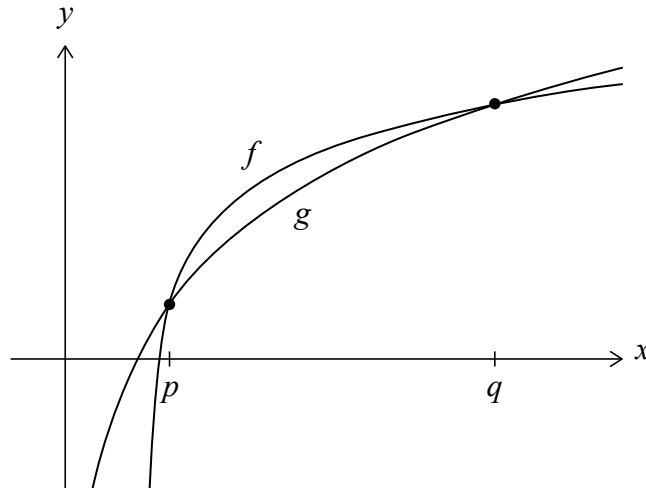
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(Question 8 continued)

The following diagram shows part of the graphs of $y = f(x)$ and $y = g(x)$.



The graphs intersect at $x = p$ and $x = q$, where $p < q$.

- (c) In the case where $d = 10$, find the value of $q - p$. Express your answer in the form $a\sqrt{b}$, where $a, b \in \mathbb{Z}^+$.

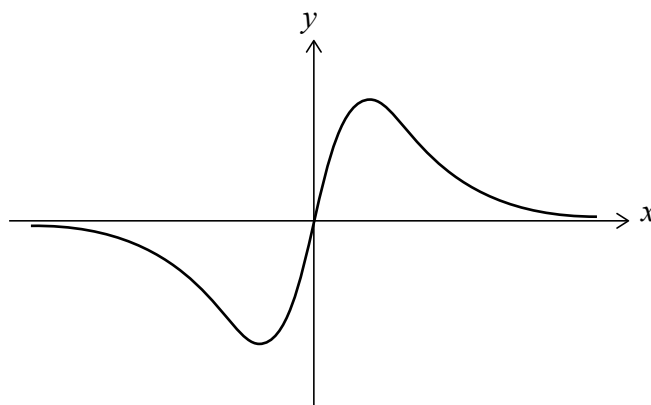
[5]



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9. [Maximum mark: 13]

Consider the function f defined by $f(x) = \frac{8x}{(x^2 + 1)^3}$, where $x \in \mathbb{R}$. The graph of f is shown in the following diagram.



(a) Show that $f'(x) = \frac{8(1-5x^2)}{(x^2 + 1)^4}$. [4]

(b) Find $\int f(x)dx$. [4]

Consider a function $g(x)$ defined for $x \in \mathbb{R}$. The derivative of g is such that $g'(x) = f'(x)$, for all $x \in \mathbb{R}$.

Let R be the region enclosed by the graph of f , the graph of g , the line $x = 0$ and the line $x = 3$. The area of R is $\frac{27}{2}$.

(c) Find the two possible expressions for $g(x)$. [5]

